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Cohort Mortality Risk or Adverse Selection in the UK Annuity Market?

DP 04/2013-017

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April 2013

Abstract

The "money's worth" measure has been used to assess whether annuities are fairly valued and also as evidence for adverse selection in the annuity market. However, a regulated life assurer with concerns about predicting long-run mortality may price annuities to reduce these risks which will affect the money's worth. We provide a simple model of the effect of cohort mortality risk on the money's worth. We demonstrate that cohort mortality risk is quantitatively important, and show that it is not possible to identify the effect of a cohort mortality risk model from that of an adverse selection model.

Keywords: Adverse selection, insurance markets, annuities

JEL codes: D4, D82, G22

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This paper has benefited from presentations at Universities of Bath, Bristol, McMaster, York (Toronto) and NHH (Bergen); the South West British Accounting and Finance Conference, Aberystwyth, 2012; the annuities workshop at Eidgenössische Technische Hochschule, Zürich, 2011; Royal Economic Society Conference, 2013; British Accounting and Finance Association Conference, 2013, and from comments from Tom Boardman, Antoine Bommier, Narat Charupat, David DeMeza, Moshe Milevsky, James Poterba and David Webb. The authors gratefully acknowledge support for this research from NETSPAR.

1. Introduction

Ever since the development of the theoretical model of Rothschild and Stiglitz (1976) identifying the rôle of asymmetric information in insurance markets, the search for empirical evidence on adverse selection has yielded conflicting findings depending on the characteristics of the particular market (Cohen and Siegelman, 2010). A common approach has been to investigate the positive correlation property, whereby higher-risk individuals buy more insurance. In the context of life annuities, higher risk corresponds to higher life expectancy and a direct test would be that individuals who have private information about their life expectancy select into back-loaded annuity products and hence individuals who buy back-loaded products live longer. However, this test is not feasible because we do not have the relevant data. An indirect test of the same phenomenon is whether life assurers recognise adverse selection and price accordingly, leading to different money's worths for different annuity products. This has been studied in a number of papers (Mitchell et al, 1999; Finkelstein and Poterba, 2002, 2004; Cannon and Tonks, 2004, 2008) who have examined the pricing of life annuities using the *money's worth* metric, defined as the ratio of the expected value of annuity payments to the premium paid.[1] Two stylised facts that emerge from this literature are that typically (i) the money's worth is less than one; and (ii) the money's worth of back-loaded annuities is less than that for front-loaded annuities.[2] For example, Table 5 of Finkelstein and Poterba (2002) (hereafter F&P) reports that in the U.K.'s compulsory annuity market the money's worth of level annuities (which pay a constant income for life) for 65-year old males is 0.900, but for real annuities

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[1] James and Song (2001) provide an international comparison of money's worth studies. Cannon and Tonks (2008, ch. 6) report further money's worth calculations for the United Kingdom, Chile, Switzerland, Australia and Singapore. Since then further analyses have been conducted for Canada (Milevsky and Shao, 2011); for Germany (Kaschützke and Maurer, 2011); for the Netherlands (Cannon, Stevens and Tonks, 2012); for Singapore (Fong, Mitchell and Koh, 2011); and for Switzerland (Bütler and Staubli, 2011). The money's worth has also been used in analysis of decision making by Fong, Lemaire and Tse (2011).

[2] In the U.K.'s compulsory purchase annuity market individuals are required to annuitise their defined contribution pension wealth, so any selection affects will be manifest through the annuity product purchased rather than the decision to annuitise, since all types must buy an annuity.

(which pay an income that is indexed to the rate of inflation) is 0.825. These two observations have been interpreted as evidence of adverse selection, that annuitants have more information about their life expectancy than insurance companies, and select into different types of annuities, which is then reflected in equilibrium annuity prices.[3]

However, in this paper we demonstrate that these facts would also be consistent with a model where there were no adverse selection and where the variation in annuity rates for different types of annuity were due to the different costs of supplying annuities. Either because life assurers are prudent or because of regulatory requirements, riskier liabilities such as real annuities have to be priced to ensure sufficient reserves are available and matched to similar real assets and these effects make them more costly. We identify three additional costs for real annuities: greater cohort risk; greater idiosyncratic risk and greater management costs. The last two additional costs could, in principle, be measured fairly easily if adequate data were available and in this paper we discuss the extent to which this is possible. The first additional cost will be shown to be more problematic. The route by which cohort risk and adverse selection affect annuity prices is the same, namely the duration of the annuity. This makes identifying the importance of the two explanations for annuity prices difficult or impossible. In this paper we quantify the costs of the risks and show they are sufficiently large to explain much of the observed variations in the money's worth, leaving a less important rôle for adverse selection.

We approach this problem by modelling explicitly the risky nature of an annuity liability. In all of the papers that we have cited the money's worth calculations are "deterministic", in the sense that it is implicitly assumed that survival probabilities for the cohort of annuitants are known. Even if a life assurer has a sufficiently large pool of annuitants to diversify away idiosyncratic risk, it still needs to forecast the future survival probabilities and such forecasts are risky. Combined with prudential or regulatory reasons to avoid downside risk this means that, the greater the risk of a liability, the greater the reserves needed by the life assurer to ensure that the

[3] Althougth DeMeza and Webb (2012) demonstrate that correlations between types of insurance contracts and risks, are neither necessary nor sufficient for the existence of asymmetric information in insurance markets.

liability can be met. In this paper we introduce the concept of the *stochastic money's worth* which takes into account the uncertainty faced by annuity providers predicting long-run mortality. We suggest that estimates of the stochastic money's worth are the appropriate risk metric for life assurers.

The attitude of public policy to the pricing of annuities is multi-faceted. Low annuity rates result in lower income streams for pensioners and the risk that they will be entitled to more means-tested benefits; as voters they may also complain more. For these reasons governments wants annuity rates to be high. On the other hand, high annuity rates correspond to higher future annuity payouts by providers for a given premium, raising questions about the providers' future solvency. In its capacity as financial regulator, the government would not want annuity rates too high if this meant that annuity providers might end up failing, because then pensioners would end up in poverty and the government would have to compensate them either through a direct rescue or by paying higher means-tested benefits.

In the United Kingdom, where the compulsory annuity market is large - worth £11 billion per year (HM Treaury, 2010b) and is an important component of pension provision, the government has faced both problems in the recent past, and continues to do so. Annuity rates have fallen consistently since 1994, which has proved politically sensitive, prompting the Department of Work and Pensions to investigate the cause of these reductions (Cannon and Tonks, 2009). In the popular press, low annuity rates have been cited as a reason for removing the compulsory annuitisation requirement in the UK.[4]

At the same time, the UK government is still dealing with the failure of Equitable Life to provide sufficient reserves for a set of guaranteed annuities sold in the 1980s. This became apparent in the late 1990s and resulted in a court case in 2000 (Equitable Life Assurance Society v Hyman) followed by the Penrose Report of 2004. The most recent "Abrahams" report (Parliamentary and Health Service Ombudsman, 2008) found that the financial regulators in the United Kingdom (initially the Department of Trade and Industry, DTI, and then the Financial Service Authority) had made errors over a ten-year period in the regulation of Equitable Life, dating from the time of the original problem in the 1990s and continuing even

[4] For example: *The Telegraph*, 12 May 2012, "Annuity rates have plunged to such low levels that they are not now a 'viable option' for millions of savers at retirement."

after the court case. Although the products sold were not conventional immediate annuities, the conclusion is clear: that maladministration by financial regulators can result in a government liability. The total cost to the UK government is expected to be £1.5 billion (H.M. Treasury, 2010a) or approximately 0.1 per cent of UK GDP.

Apart from the Equitable Life débâcle and before the financial crisis of 2007, the regulator was encouraging life assurers to price conservatively. For example, the chairman of the Financial Services Authority wrote to life assurers recognising that companies would usually make assumptions based on their own mortality experiences, but adding

> *"...if this is not possible we would expect firms to consider the different industry views in this area and to err on the side of caution."* (FSA Dear CEO letter, April 2007)

A further motivation for appropriately assessing the risks associated with annuities is the proposed EU-wide changes to insurance regulation enshrined in *Solvency II*, which will take effect from 2013. Solvency II applies to the insurance industry the risk-sensitive regulatory approach adopted in the Basel 2 reforms for the banking industry. Under the proposal for Solvency II, life insurance companies are required by the regulatory framework to allow explicitly for uncertainty in their valuations:

> *"the technical provision under the Solvency II requirement is the sum of the best estimate and the risk margin, . . ., the best estimate is defined as the probability-weighted average of future cash flows . . . The probability-weighted approach suggests that an insurer has to consider a wide range of possible future events: for example, a 25% reduction in mortality rates may have a small probability of occurrence but a large impact on the cash flows. However, the assumptions chosen to project the best estimated cash flows should be set in a realistic manner, whereas the prudent allowance for data uncertainty and model error should be taken into account in the risk margin calculation."* (Telford et al, 2011; paras. 7.2.1 - 7.2.2.3).

As is made explicit in text books such as Booth et al (2005), actuaries take risk into account when pricing annuities. To gain some idea of the magnitude of this issue in the UK, we may examine the life assurers' FSA Returns. As a regulated industry, each life assurer must declare the actuarial assumptions used to value its liabilities, by comparing the mortalities used in its own calculations with the mortalities in the benchmark tables produced by the Institute of Actuaries' Continuous Mortality Investigation. The CMI collects data from all of the major life assurers, aggregates and anonymises it and then analyses the pooled data. So the CMI tables of mortality approximate to the average mortalities across the whole industry. The figures presented in life assurers' FSA returns are then compared to this average and we summarise the figures for the major annuity providers in Table 1 and illustrate them in Figure 1.[5]

[Table 1 about here]

[Figure 1 about here]

In investigating annuity pricing, one of the problems for researchers is knowing which benchmark series is appropriate. F&P (2002) used the "life office pensioner" mortality table, which reports mortalities of members of occupational defined-benefit pension schemes administered by life assurers: the most recent version of this table is PCMA00, which is very similar to the series for DC pensioners. Since annuitants are defined-contribution (DC) pensioners, this is arguably the wrong series, but the more relevant DC personal pensioners described by PPMC00 is based on a relatively small sample, so we prefer to follow F&P (2002).

Figure 1 shows that up to age 67, only Canada Life assumes higher mortality than the benchmark: for ages greater than 68, every life assurer assumes lower mortality rates than the benchmark. So every life assurer is assuming that their annuitants' mortality is lower than the average (and life expectancy is greater than average). Some of the variation in assumptions between companies must be due to genuine variations in mortality of the annuitants, but it is obviously impossible that every company has lower mortality than the average. This is *prima facie* evidence that firms are building some allowance for mortality risk into their valuations.

[5] The CMI tables include four benchmark life tables for different annuity groups: PCMA00, RMC00, RMV00 and PPMC00. PCMA00 reports the mortalities of members of occupational defined-benefit pension schemes administered by life assurers; RMC00 and RMV00 summarise the mortality evidence of the original DC pensions - retirement annuity contracts for self-employed workers; RMV is for pensioners in receipt of a pension ("vested") and RMC is for both pensioners in receipt of a pension and for those still making contributions ("combined"); and PPMC00 reports mortalities of DC personal pensioners. Using a different benchmark would not affect our conclusions.

In this paper we propose a new approach to analyse the effect of risk on annuity pricing. Life assurers are typically able to hedge their annuity liabilities with assets such as government or very high quality corporate bonds.[6] RPI-linked annuities are backed by index-linked bonds, predominantly issued by the government.[7] It is difficult for life assurers to match the duration of their assets precisely with the duration of their liabilities, so there are small additional costs to managing cash flow: these may be bigger for RPI-linked annuities where the market for bonds is smaller. Annuity providers are able to diversify away the idiosyncratic annuity mortality risk by selling a large number of annuities to a heterogeneous population. However, the remaining large risks faced by a life assurer relate to the cohort mortality risk of the group of annuitants born in a particular period (Telford et al, 2010). This cohort mortality risk cannot be diversified away, and instead under prudential risk management life insurers reserve assets to allow for the possibility of decreases in cohort mortalities. We introduce a stochastic money's worth metric, and compute the distribution of the present value of annuity payments that allows for uncertainty in these cohort mortalities.

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[6]Any company providing long-term life assurance business must provide detailed accounts to the regulator referred to as the FSA Returns. Where investing in corporate bonds results in a higher yield (a risk premium), life assurers are not allowed to use this to value their liabilities. For example, see the note in *Norwich Union Annuity Lim*ited, Annual FSA Insurance Returns for the year ended 31st December 2005 (page 53): "In accordance with PRU 4.2.41R, a prudent adjustment, excluding that part of the yield estimated to represent compensation for the risk that the income from the asset might not be maintained or that capital repayments might not be received as they fall due, was made to the yield on assets." The return goes on to say that AAA-rated corporate bonds had yields reduced by 0.09 per cent, A-rate by 0.32 per cent and commercial mortgages by 0.41 per cent.

[7] In the U.K., where annuities are sold that are adjusted to inflation, it is possible to hedge indexed annuities by purchasing government bonds that are indexed to the same price index (i.e., the Retail Price Index, or RPI). The FSA Returns make explicit that the different types of annuities are backed by different assets. For example, the note in *Norwich Union Annuity Lim*ited, Annual FSA Insurance Returns for the year ended 31st December 2005 (page 50): "Non-linked and index-linked liabilities are backed by different assets and hence have different valuation interest rates."

It is possible to quantify mortality risk through stochastic mortality models such as that of Lee and Carter (1992), which allow us to estimate the probability distribution of future mortality. The traditional risk-neutral approach to pricing annuities is to set the price equal to the expected present value of the promised annuity payments, yielding a money's worth equal to unity (or slightly less than unity after allowing for the costs or loadings associated with the annuity provision). The Solvency II / Basel 2 approach often uses the Value-at-Risk (VaR) as a guide to suitable reserving, so a Regulator could use our estimates of the distribution of annuity values to calculate VaRs and examine the effect that VaR pricing would have on conventional measures of the money's worth. By VaR pricing we mean that insurance providers price off the tail of probability distribution of future mortality such that there is a 95 per cent chance of having sufficient assets to meet the actual risky liabilities. This approach allows us to answer two questions: First, given our estimates of the risks to life assurers, how will these estimates affect the money's worth and how the money's worth will change when yields change? Second, what are the consequences of these estimates for the pricing of different annuity products (e.g. real versus nominal) on the money's worth?

We start by describing in section 2 the conventional money's worth measure and how it is calculated in practice. We then briefly review the theory of adverse selection in the annuity market in section 3 and discuss whether it is appropriate to characterise the market as being a separating equilibrium. In section 4 we review the evidence for the money's worth in the UK. In section 5 we show how a probability distribution of the value of an annuity can be constructed using a stochastic mortality model. We use this to measure the risk for annuities and the consequences when a researcher calculates the money's worth based on a deterministic projection of mortality while annuity providers are pricing to take into account the financial risk associated with mortality risk and a given set of interest rates.

2. Money's Worth Calculations

2.1 The (Conventional) Money's Worth

The conventional measure of the value of an annuity is the money's worth (Warshawsky, 1988; Mitchell et al, 1999), which compares the expected present value of the annuity payments with the price paid for the annuity. We define the annuity rate A as the ratio of annual payments to the actual purchase price of an annuity.[8] Then the money's worth for a 65-year old would be

(1) $$\text{Money's Worth} \equiv \frac{A_{t,65}}{12}\sum_{x=65}^{\infty} R_{t,(x-65)} E_t\left[s_x\right] \qquad s_x \equiv \prod_{i=0}^{x-1} p_{t+i,65+i}$$

where $p_{t+i,65+i}$ is the one-period survival probability for the annuitant who is age $65+i$ in period $t+i$ (that is the probability of living one more period conditional on being alive at the beginning of the period) and s_x is thus the probability of the current 65-year-old living to age x or longer. It is conventional in money's worth calculations to try to use the survival probablity actually in use at time t rather than the *ex post* survival probabilities of the annuitants and hence the money's worth is written in terms of $E_t\left[s_x\right]$ rather than s_x.

The discount factor $R_{t,j}$ is usually inferred from the yield curve on government bonds and it is assumed that this rate of return is risk free. As with the survival probabilities, the yields used are those in force at the time of the sale of the annuity rather than returns which might have been available thereafter.

There are two possible justifications for using government yields. The first is that annuity payments are meant to be secure (i.e. the chances of default are minimal) and the interest rate on government bonds is typically the best possible guess at the "safe" rate of interest. For some countries the interest rate on government debt would not be risk free, but this seems a reasonable approximation for countries such as the U.K. for which debt is typically AAA rated. Where life assurers use commercial bonds (or commercial mortgages), they must adjust the higher rates of return for the greater risk and, to a good approximation, the risk-adjusted rates of return on commercial bonds or mortgages are likely to be the same as the rates of

[8] For example, in the UK in July 2009, the Prudential would sell an annuity for £10,000 to a 65-year old man which would pay a monthly income of £61, or £732 annually for life: the annuity rate would be $A_{2009,65} = 732/10,000 = 0.0732 = 7.32\%$.

return on government bonds.[9] Secondly, life assurers approximately match their annuity liabilities with government bonds and regulation may even compel them to do so.[10]

2.2 Survival Probabilities for the (Conventional) Money's Worth

Equation (1) requires us to know the probability of living $p_{t+i,65+i}$, or equivalently the probability of dying, referred to as the mortality.[11] The estimation of these variables is a staple of actuarial textbooks (Bowers *et al,* 1997; Pitacco et al, 2009), but forecasting these variables is more problematic and usually relies on extrapolating the past trend, since models based on the causes of death are insufficiently precise to be used for prediction purposes. The resulting estimate is subject to uncertainty from a variety of sources.

The estimates will be based on data available up to time *t* (or possibly earlier if there are lags in data collection): other than measurement errors, there is also the problem that death rates are not quite the same as death probabilities and there may be considerable sampling error if the death rates are based on relatively small samples (which is often the case for the highest ages). There may be additional changes in the data generating process either because the health of annuitants changes relative to that of the population as a whole or because the health of pensioners is different from others and pension coverage changes - these are two different forms of a selection effect. In many countries sufficiently detailed data for *p* are simply unavailable and the U.K. is unusual in having reliable data for pensioners over a long time period: since 1924, U.K. life offices have provided their firm-level data to a central committee of actuaries who anonymised and pooled this

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[9] Details of the notional yields, credit ratings and corresponding adjustments are reported in the FSA returns. Price risk is relatively unimportant since bonds are typically held to maturity.

[10] CGFS (2011) provides a review of international insurance regulation and notes that this matching can be *duration matching* which only partially matches liability and asset cash flows and *cash-flow matching* which perfectly matches the flows. The footnotes of various FSA returns note that perfect matching is impossible and that there is a small residual risk. This matching is likely to be more difficult for RPI-linked annuities and we return to that issue in section 3.4.

[11] More formally, mortality μ is the continuous-time analogue of the one-year death probability $q \equiv 1 - p = \int d\mu$.

information to create a large enough data set to enable reliable statistical analysis and long-term projections. Until 1999 (i.e. the "92" series) the projections were only updated infrequently. The "interim adjustments" to this series in 2002 allowed for three different scenarios and the "00" series of 2006 did not even attempt to project mortality into the future but simply described the evolution of the data up to 2002. When we estimate the money's worth we use the contemporaneous benchmark tables and projections.

Second, there is model uncertainty. Most models' starting point is Gompertz's Law, i.e. the empirical regularity is that the logarithm of death rates tends to increase approximately linearly. The caveats to this are that: the decline is only approximately linear; the speed of decline depends upon age; and there are occasional structural breaks, which may apply either to the whole population or to just some cohorts. There is also some doubt as to whether one should look at the logarithm of the death rate or a logistic function and whether the decline is a stochastic or deterministic trend (Cairns *et al*, 2009).

2.3 Estimates of the Money's Worth

Figures 2 and 3 illustrate respectively our monthly annuity rate data for 65-year old men in the UK compulsory purchase market[12] and the money's worths for annuities for three different ages (65, 70, 75). To allow for the fact that mortality projections were changing during this period, we calculate the money's worth using the relevant mortalities for each period from the *Institute of Actuaries* (the annuity rate data and interest rate data are the same throughout), with a small overlap to allow for uncertainty over the precise point at which a new table should be introduced. Each new actuarial table tends to result in an increase in the money's worth due to longer projected life expectancy, but the medium cohort projection and the PNML00 projection match almost exactly. The evidence in Figure 3 suggests that there was a fairly small decline in money's worths for a male aged 65, but little change for males aged 70 or 75: in fact the range of money's worths fell considerably.

[Figures 2 and 3 about here]

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[12] The data are discussed in more detail in the appendix.

Comparing Figures 2 and 3, it is apparent that the decline in annuity rates of about 2.5 per cent between 1994 and 2000 does not correspond to as large a change in the money's worth: this politically sensitive fall is predominantly explained by falls in interest rates and increases in life expectancy. The much more notable change is the apparent reversal of money's worths by age in the period from 2001 to 2004: from 2004 onwards the ordering of money's worths by age returns to the pattern found in F&P (2002), but the range is very much smaller.[13] Table 3 provides formal tests of the differences in money's worths over the four sub-periods of our data with respect to the relevant actuarial life table. In Panel A of Table 3 we compute the average money's worth by age, and examine whether there are significant differences between the money's worths of annuities at different ages. We test for the equality of means of these series, using a "matched pair" analysis to deal with trends in the series. We take the difference between any two series and evaluate the autocorrelation function to choose appropriate lag length. We then calculate the t-statistic for the mean value of these differences, using Newey West standard errors, with the relevant adjustment for the autocorrelation structure. The reversal of the money's worth by age over the period 2001-2004 for 70-year old males (t-stat on difference with 65-year old males is -1.94) is inconsistent with the suggestion of F&P (2002) p. 41 that lower money's worth at higher ages is evidence for asymmetric information.

[Table 3 about here]

[Figures 4 and 5 about here]

Figure 4 and Panel B of Table 3 shows the money's worth for annuities with guarantee periods, although there is little difference in the money's worths of these guarantees. Figure 5 and Panel C of Table 3 shows the money's worths for level, real and escalating annuities: we are able to confirm the findings of F&P (2002) that back-loaded annuities (real and escalating) have significantly lower money's worths (0.768 and 0.802) than level annuities (0.859). Other than the fact that the money's worth tends to be lower than in F&P (2002), the qualitative behaviour of level annuities versus escalating or real is similar. Comparing the beginning of the period to the end (the two periods when we are relatively confident about the appropriate

[13] Cannon, Stevens and Tonks (2012) analyse the Dutch annuity market and also find an inverse pattern of money's worths by age for the period 2001-2010.

mortality table to use), there is some slight evidence that the money's worth has fallen and that the gap between the nominal and real money's worth has risen. The results for the relative money's worths of real and escalating are more mixed: the gap between them is often small and sometimes the money's worths of real annuities is slightly higher than for escalating, rather than lower.

Overall, our analysis for the money's worth over the whole period largely confirms that of F&P (2002). The caveats are that the differences in money's worth by age or guarantee period have disappeared by the end of the period. However, we now suggest an alternative interpretation to F&P on explaining the pattern in these money's worths.

3. Adverse selection in Annuity Markets

3.1 A simple model of adverse selection

To formalise the implications of asymmetric information for the money's worth we need to consider a model of adverse selection, similar to Rothschild and Stiglitz (1976), hereafter R&S. Agents live for up to two periods: all agents live in period one for certain and into period two with a survival probability that depends upon an annuitant's individual characteristics. Suppose that there are two types of annuitant, distinguished only by their survival probabilities, which are p^h for high-life-expectancy individuals and p^l for low-life-expectancy individuals, where $p^h > p^l$. A high life expectancy individual corresponds to a high risk from the perspective of the life assurer. The probabilities p^h and p^l are common knowledge, but life assurers do not know annuitants' types.

In the UK compulsory purchase market, individuals must annuitise their pension wealth.[14] F&P (2002, 2004) suggest that an adverse selection separating equilibrium could be achieved through agents with different life expectancies buying different products, since product type is the only choice open to a personal pensioner. Agents maximise respectively:

$$U^h = u\left(c_1^h\right) + p^h \delta u\left(c_2^h\right)$$

$$U^l = u\left(c_1^l\right) + p^l \delta u\left(c_2^l\right)$$

[14] We discuss the details of this assumption in section 3.3.

We also assume that agents have the same felicity function $u(\bullet)$. Note that because there is no savings, consumption is the same as the annuity payment. Each agent has pension wealth normalised to one. In period 1 an agent purchases an annuity and immediately receives the first payment a_1 ; if the agent survives to period 2 then there is a second payment $a_2 = \phi a_1$, where ϕ allows us to characterise an annuity as front-loaded, $\phi < 1$, or back-loaded, $\phi \geq 1$.

In this section, we assume that life assurers are risk neutral and price annuities actuarially fairly, so the expected-break-even condition for each type in the separating equilibrium is

$$(2) \qquad 1 = a_1^i + p^i \frac{a_2^i}{(1+r)} \Rightarrow a_1^i = \frac{1+r}{1+r+p^i\phi}$$

where *r* is the wholesale interest rate earned by the life assurer on assets matching the annuity liability. If an annuity makes payments in nominal terms *r* is the nominal interest rate and if the annuity makes payments indexed to the price level then *r* is the RPI-indexed (real) interest rate.

[Figure 6 about here]

The budget constraints and indifference curves are plotted in Figure 6, which looks like the standard diagram for the R&S model of insurance. However, there are subtle differences: first, whereas in the R&S model the two axes show consumption in the two different states of the world, in this diagram the vertical axis shows the certain consumption in period one and the horizontal axis shows how much the individual consumes conditional on surviving into period two; second, at any point on the budget constraint an annuitant is fully insured in the sense that he will not outlive his resources (the only way to be under-insured against longevity risk is to invest some wealth in non-annuity form, which cannot be shown in this diagram). So different points along a budget constraint do not show different levels of insurance but different consumption paths through time: for example point A has the same level of consumption in both periods (e.g. a real annuity) whereas point B has a higher level of consumption in period one (e.g. a nominal annuity during a period of inflation) at the expense of lower consumption in period two.

In an R&S separating equilibrium, a life assurer would offer two contracts, at A and B. High-life-expectancy annuitants would choose a back-loaded contract A, whereas low-life-expectancy agents would choose a front-loaded contract B. The diagram illustrates the requirements necessary for a separating equilibrium to exist: the two types of annuitant must have indifference curves with different slopes and it must be possible to offer a contract such as B, where consumption in period one is higher than consumption in period two. How much consumption must be higher in period one than period two depends upon the convexity of the high-life-expectancy individual's indifference curve. The separating equilibrium is achieved through agents' inter-temporal substitution of consumption rather than variations in the amount of insurance purchased.

We can now determine the consequences of these equilibria for the money's worths of the two contracts that would be calculated by a researcher. If information were available on survival probabilities of both high- and low-risk annuitants then the money's worths for both contracts should equal one (from the fact that the contracts are actuarially fair). In reality, the only survival probability available to the researcher is that provided by the CMI for the whole pool of annuitants. This observed survival probability is

(3) $$\overline{p} = \theta p^{l} + (1-\theta)\, p^{h}$$

where θ is the proportion of the annuitant population which is low-life expectancy and the researcher only has access to $\overline{p}$, not information on p^{l} or p^{h}. The researcher might have some information about θ. For example, Table 2 shows the number of annuities sold by leading life assurers in the UK taken from the FSA returns, which require life assurers to distinguish sales of nominal and real annuities: clearly the proportion of real annuities sold is tiny. If it were really the case that only low-life-expectancy annuitants purchase level annuities, then it would follow that $\theta \approx 0.99$.

[Table 2 about here]

Using the information available the researcher calculates the money's worth for each type as

(4) $$MW^{i} = a_{1}^{i} + \overline{p}\frac{a_{2}^{i}}{(1+r)}$$

The high-life-expectancy annuitant chooses contract A where the annuity payments are the same in both periods $\phi = 1$, so

(5) $$MW^h = \frac{1 + r + \overline{p}}{1 + r + p^h} < 1$$

and the money's worth for the low-life expectancy annuitant (who buys a front-loaded annuity with $\phi < 1$) is

(6) $$MW^l = \frac{1 + r + \phi\overline{p}}{1 + r + \phi p^l} > 1 > MW^h$$

which confirms that the money's worths calculated by a researcher should appear to be greater for low-life-expectancy individuals than for high-life expectancy individuals. Although equation (6) suggests that $MW^l > 1$, the fact that $\theta \approx 0.99$ means that in this model it will actually be very close to one. The fact that it is virtually never that high suggests a rôle for administrative costs in explaining the money's worth.

3.2 Forecast rather than known survival probabilities

In this section we consider a simple model of cohort mortality risk where there is no annuitant heterogeneity (and hence no adverse selection), but the life assurer must reserve against cohort mortality risk, ensuring that it acts as if risk averse. This assumption is a major difference from the risk neutrality assumed in the previous section which determined the break-even condition of equation (2). A simple way to characterise this risk aversion, consistent with Solvency II regulations, would be to assume that the life assurer uses a Value-at-Risk method based on an appropriate percentile of the perceived distribution of cohort survival probabilities so that the new break-even annuity price equation is

(7) $$1 = a_1 + \tilde{p}_{C\%}\frac{a_2}{(1 + r)}.$$

where $\tilde{p}_{C\%}$ is the appropriate centile from the distribution of projected probabilities.

Since there is no adverse selection, the only differences are between types of annuity product: level, escalating and real. From equation (7) the resulting money's worth calculated by a researcher for each product would now be

(8) $$MW = \frac{1+r+\bar{p}\phi}{1+r+\tilde{p}_{C\%}\phi} < 1 \quad \text{since } \bar{p} < \tilde{p}_{C\%}$$

We can draw three inferences from this equation across product types. First, all annuity types would have a money's worth less than one. Second, since $\partial MW/\partial\phi < 0$, escalating annuities with $\phi > 1$ will have a lower money's worth than level annuities with $\phi = 1$. Third, since $\partial MW/\partial r > 0$ real annuities will have a lower money's worth than level annuities (although the relationship between escalating and real annuities is ambiguous).

A further problem with real annuities is that the number of annuitants is much smaller, as we have seen in Table 2. A consequence of this is that sales of real annuitants suffer not only from cohort mortality risk but also idiosyncratic mortality risk. This means that $\tilde{p}_{C\%}^{\text{Real}} > \tilde{p}_{C\%}^{\text{Nominal}}$, which will reinforce the difference between the MW^{Real} and $MW^{Nominal}$.

This simple model suggests that observed pattern of money's worths may be due to pricing annuities to account for cohort mortality risk rather than due to adverse selection. In section 4 we will undertake simulations to assess whether the magnitude of reserving is significant. Before those calculations we make a few additional points about the money's worth.

3.3 Additional comments on the plausibility of the adverse selection model and the rôle of administrative costs

In this section we make some qualitative remarks about the adverse-selection separating-equilibrium characterisation of the annuity market and the issue of costs. Discussion of adverse selection in insurance markets normally centres on whether agents purchase full or partial insurance. In the UK it is virtually obligatory for a personal pensioner to purchase an annuity with at least 75 per cent of the value of the pension fund (up to 25 per cent can always be taken as a tax-free lump sum). In principle it is possible to avoid annuitisation until age 75 and even then one can avoid annuitising all of one's pension wealth. In practice deferring or avoiding annuitisation is unattractive for most individuals, except perhaps the very wealthy who are doing so for complicated tax reasons. If one defers annuitisation until 75 one must enter "drawdown", which is expensive and limits the amount of the fund that can be accessed. In particular this would severely limit the possibility for individuals who believed they had short life expectancy to bring forward consumption. To avoid annuitisation one must first demonstrate that one has a secure pension income (perhaps purchased from a portion of the pension fund) and then pay 55 per cent tax on that part of the pension fund which is not annuitised: the 55 per cent tax effectively reclaims all of the tax privileges that a higher-rate tax payer would have received from saving in a pension fund.[15]

For these reasons we can treat annuitisation of personal pension wealth as effectively compulsory for nearly all personal pensioners and the only choice available is the timing of annuitisation and the type of annuity purchased. Evidence from the Association of British Insurers shows that the vast majority of annuities are purchased at the conventional retirement ages of 55, 60 or 65, suggesting that timing is driven almost entirely by retirement (very few annuities are purchased at age 75, further evidence that the option to defer annuitising until then is not very important).

Despite the compulsory nature of annuitisation in the U.K., life assurers are unable to observe the consumption choices of the annuitants and the annuity payments may be very poor proxies for consumption. Most annuitants will have wealth other than their personal pension wealth which will not be observed by the life assurer. At a minimum, annuitants are likely to have the UK's Basic State Pension, plus additional means-tested benefits derived from the Minimum Income Guarantee. Some annuitants will own a house (non-annuity wealth): those without housing wealth will receive Housing Benefit (since this will either be received until death or superceded by long-term care assistance, it is close to annuity wealth). On top of this, some annuitants will also have occupational pensions and other personal pensions: it is possible to have more than one personal pension fund and not necessary to combine them at the point of annuitisation.[16] Finally, in section 3.1 we assumed that no savings was allowed outside the annuity, but it would be possible

[15] The details of compulsory annuitisation rules have changed several times in the last decade. The most recent rules are described in HM Treasury (2010b). Earlier variants are discussed in Cannon and Tonks (2009).

[16] Data are not yet available for researchers to be sure of the distibution of different pension funds across pensioners and life assurers would not have this information.

for a high-life expectancy annuitant to buy a nominal annuity and save a proportion of the annuity payments for consumption in period two.

For all of these reasons, a life assurer is only able to observe a_1/a_2 on what may be a relatively small part of an annuitant's total wealth but not c_1/c_2 which is what is needed to effect an R&S separating equilibrium.

Further, contract B in Figure 6 may not be allowed by regulators. Annuities in the compulsory purchase market have to be recognised by the tax authorities (HMRC). In practice the types of annuity allowed are level (constant in nominal terms), RPI-linked (constant in real terms) and escalating (nominal payments rising at (typically) 3 or 5 per cent per year). In a low-inflation environment, a level annuity does not allow for much front-loading: translated to Figure 6 it might be that the most heavily front-loaded contract available is at point D. Unfortunately, offering contracts A and D will not result in a separating equilibrium. A corollary of this is line of reasoning is that it will be easier to achieve a separating equilibrium in a high-inflation environment, but the variation in inflation is too small during our period of observation to attempt to use this result.

We make one final comment on the effect of life assurers' administrative costs. Life assurers are involved in many sorts of insurance and long-term fund management and there are presumably large economies of scope. This means that it would be very difficult to allocate precisely all of the relevant costs to a firm's annuity business alone, let alone the costs of particular types or cohorts of annuitants. However, as noted by F&P (2002), it is probable that the costs of managing real annuities are higher than for level annuities. HM Treasury and Bank of England (1995) describe several reasons why the market for RPI-indexed bonds is thinner and less liquid than for conventional bonds and the differences are considered sufficiently important that the bonds are issued in different types of auction (Debt Management Office, 2013). Because fewer RPI-indexed bonds are issued this will almost certainly make complete cash-flow matching more difficult. However, we are unable to quantify the cost of this.

4. The Stochastic Money's Worth

In the previous section we showed how the pattern of observed money's worths might be due to life assurer's reserving against cohort mortality risk. In this section we quantify this effect. This requires us to quantify the uncertainty in forecasting mortalities or, in the notation of section 3, the uncertainty in forecasting *p*. There are two components to this: first, one has to know that one has the correct data, the correct model and to be sure that the forecasting method will not be compromised by structural breaks; and second, given the previous considerations one has to have a stochastic model. We shall be ignoring all of the first set of considerations: we are using life office pensioner data (rather than personal pensioner data); from the array of potential models (e.g. described in Cairns et al, 2009), we simply choose a model which is widely used and understood; the period after our data end was characterised by significant changes in models and forecasts due to the perception of structural breaks or cohort effects. By concentrating on the uncertainty within a particular model we are under-estimating the effect of uncertainty on the money's worth.

The model we use for this exercise is that of Lee and Carter (1992) model, which has been widely accepted as a starting point for mortality analysis.[17] The LC model has a flexible (non-parametric) relationship between age and log-mortality which is assumed to be constant: projection of mortality consists of simple shifts in the log-mortality-age curve. More specifically, the one-year death probabilities are modelled as

(9) $$\ln(1-p_{xt}) = \ln q_{xt} = \alpha_x + \beta_x \kappa_t + \varepsilon_{xt} \qquad \varepsilon_{xt} \sim \mathcal{N}(0,\sigma^2)$$

which can be estimated by least-squares from a singular-value decomposition method (see Pitacco et al, 2008; Girosi and King, 2009, for an exposition).[18] There are a variety of identification estimation issues which we discuss in Appendix B. Regardless of the estimation procedure, forecasting is based upon

(10) $$\Delta\kappa_t = \lambda + \psi_t \qquad \psi_t \sim iid\left(0,\sigma_\psi^2\right)$$

[17] As a robustness check to our analysis we consider an alternative to the Lee-Carter approach: the Cairns-Blake-Dowd (2006) model, which builds on the empirical observation that the relationship between log-mortality and age is approximately linear, and uses this as a restriction in the estimation strategy.

[18] It would be possible to estimate the LC model using maximum likelihood rather than least squares, but throughout most of our period LC models were estimated by least squares.

where the parameters λ and σ_ψ^2 are estimated in a second-stage regression (and where a more complicated dynamic process than a random walk is also possible).

A simpler procedure is to ignore the fact that mortality is following a stochastic trend: Girosi and King (2009) suggest that it is common in practice to project the κ_t terms using

(11) $$\Delta\kappa_{t+s} = \lambda s$$

which ignores the fact that the κ_t terms will be evolving randomly. We shall refer to this as the *deterministic projection* of the Lee-Carter model. Alternatively, additional forms of uncertainty can be included in the model: for example the parameters λ and σ_ψ^2 must themselves be estimated and we shall incorporate estimates of the parameter uncertainty in our estimates of $\hat{\lambda}$ and $\hat{\sigma}_\psi^2$, referred to as a model with *uncertain parameters*.

Our estimate of the Lee-Carter model uses the UK's life office pensioner mortality data, which is the largest and most commonly used data for UK private pensions. The data we use are for 1983-2000:[19] the typical exposed to risk for a given age in a given year is in the range 5,000-10,000, although there are fewer for very high ages. The total exposed-to-risk in 1983 is 356,552 and in 2000 it is 289,019. This period, and the years immediately following it, demonstrated significant falls in mortality, requiring substantial revision to life tables as documented in Cannon and Tonks (2008, section 6.2).[20] Consistent with Gompertz's law the alphas and betas are approximately linear in age, and the kappa is a stochastic trend. The fact that beta depends upon age shows that the trend in log-mortality is age dependent.

Using the estimated alphas and betas and with projected kappas, we can project survival probabilities into the future using numerical methods (we conduct a Monte Carlo with 100,000 replications). Figure 7 shows the survival fan chart for a male

[19] Although detailed data on pensioner mortality were collected in the United Kingdom from 1948 the data prior to 1983 have been lost (CMI, 2002).

[20] In this data set no 60-year old male died in 1998, so the log mortality was not defined: we replaced the zero value by 0.5 (which corresponded to the lowest mortality rate observed elsewhere in the data set). A variety of alternative assumptions resulted in almost identical conclusions.

aged 65 at the end of the period of our data in 2001. Such fan charts have been discussed in Blake, Dowd and Cairns (2008): there is relatively little uncertainty about the survival probability for the first few years: the probability of dying is small and there is little scope for uncertainty. However, by age 75 there is considerable uncertainty. Note that an annuity which was more back-dated (had longer duration) would have a higher proportion of its present value paid in the period of greater uncertainty and thus would be a riskier liability for a life assurer.

[Figure 7 about here]

Using the distribution of survival probabilities from Figure 7 we then estimate the distribution of the value of an annuity paying £1 per year and illustrate this in Figure 8 for different interest rates, assuming that the yield curve is horizontal (the same interest rate at all terms). As expected, Figure 8 shows that, as the interest rises and the duration of the annuity falls, both the expected value of an annuity and the standard deviation fall.

[Figure 8 about here]

Table 4 shows the consequences for the money's worth if a life assurer prices annuities from the relevant centile of the distribution of annuity values but the researcher uses the expected annuity value. When priced from the median, the money's worth is approximately one, since the median and expectation are virtually the same. When the life assurer prices from the 99th centile, the money's worth is less than one and the discrepancy is larger the lower the interest rate (since the duration of the annuity rises and is where there is greater uncertainty).

The left-hand panel of the table shows the effect when there is no idiosyncratic mortality uncertainty: the figures are based purely on the uncertainty in the distribution of projected cohort mortality uncertainty. The right-hand panel quantifies the effect of idiosyncratic uncertainty that would be faced by a life assurer who sells only 400 annuities: in each Monte Carlo replication the future one-year death probabilities are generated for all ages and then the actual number of deaths are drawn from a Bernoulli distribution with the projected probability).

[Table 4 about here]

We can now use the table to quantify the possible effect on the money's worth. Suppose the nominal interest rate were 9 per cent and the real interest rate were 4 per cent, figures roughly consistent with the ten-year government bond yields of 1994 in Figure 2 and a life assurer were pricing off the 95th centile (i.e. a VaR of 95 per cent). Then the money's worth for nominal annuities would be 0.969 (from the left hand panel of the table) and the money's worth for real annuities would be 0.934 (from the right hand panel of the table), a difference of 3.5 per cent. Slightly more than half of this arises from the real interest rate being lower than the nominal rate and the rest arises from the imperfect diversification of having only 400 real annuitants. If the life assurer were pricing from the 99th centile, then the difference would be just over 5 per cent. Despite being an under-estimate of the effect of cohort mortality risk due to ignoring model uncertainty and ignoring additional costs of real annuities, this is about half of the observed difference in the money's worth in 1994 from Figure 5.

In Figure 9 we illustrate our final calculations making use of the actual interest rates that were used in the money's worth calculations in Figures 2-4. Notice, however, that we are using a constant set of mortality projections for the whole period, so our results are not directly comparable with the earlier graphs. Instead, Figure 9 isolates the effect that actual interest rate changes would have had on money's worth calculations had annuities been priced on the 95th centile (all calculations are for a male aged 65 and we make no allowance for idiosyncratic mortality risk). Figure 9 reinforces our calculations in Table 4: a significant part of the difference between nominal and real money's worths could be due to cohort risk.

[Figure 9 about here]

In our discussion of Figure 5 we noted that there appeared to be a slight fall in the nominal money's worth between 1994 and 2012 and that the gap between the real money's worth and the nominal money's worth had risen. Both of those features are also evident in Figure 9: this arises from the fall in interest rates which not only reduces the expected value of annuity payments (an effect capture in money's worth calculations) but also increases the uncertainty (which is not captured in money's worth calculations).

The striking difference between Figures 5 and 9 is the relative behaviour of real and escalating annuities, since escalating annuities have a consistently lower money's worth than real annuities in Figure 9, but the reverse is true in Figure 5. The empirical finding is equally problematic for the adverse selection model of annuities, which would make the same qualitative predictions as our model (as noted by F&P, 2002). Although not reported in the FSA Returns, we believe from discussions with practitioners that the number of escalating annuities is similar to the number of real annuities and therefore the difference cannot be due to idiosyncratic risk. This underlines the fact that administrative costs of real annuities have to be significantly higher than for nominal annuities for any model to fit the observed money's worths.

5. Summary and Conclusions

In this paper we have updated our money's worth calculations for the UK compulsory purchase market – the biggest annuity market in the world – to 2012. This provides a starting point for re-visiting the idea that there is adverse selection in the annuity market, following the analysis of F&P (2002). Some of their corroboratory evidence, such as the money's worth varying by age or guarantee period are no longer valid. However, their most important result, that back-loaded annuities have a lower money's worth than front-loaded annuities is still true in the UK annuity market.

Finkelstein and Poterba's explanation for this was that there is adverse selection and that a separating equilibrium is achieved via longer-lived individuals purchasing back-loaded annuities. When calculating the money's worth using the mortalities of all annuitants pooled together (i.e. the only mortality data that are available), this would result in back-loaded annuities having a lower money's worth.

In this paper we show that an alternative model yields exactly the same qualitative conclusions. Our model relies upon the fact that life assurers need to reserve against the uncertain evolution of cohort mortality, both for prudential reasons and because they are required to do so by government regulation. Because back-loaded annuities have a higher proportion of payouts in the more distant future, they are inherently riskier products and require greater reserves.

Because our model yields the same conclusions as the Finkelstein-Poterba model it is impossible to identify the magnitude of the two effects from the data alone. To address this problem we have quantified the importance of cohort mortality risk using the Lee-Carter model, although arguably this might under-state the effect as we are effectively ignoring the issue of model uncertainty. Our results suggest that a substantial proportion of observed differences in money's worths for different annuity products may be due to the relative risk. Combined with other costs of annuity supply, which are conventionally ignored in money's worth calculations, this suggests a much smaller effect for adverse selection.

References

Blake, D., Dowd, K., and Cairns, A.J.G. (2008) "Longevity Risk and the Grim Reaper's Toxic Tail: The Survivor Fan Charts." *Insurance: Mathematics and Economics, 42,* pp. 1062–1066.

Booth, P, Chadburn, R., Haberman, S., James, D., Khorasanee, Z., Plumb, R.H., and Rickayzen, B. (2005) *Modern Actuarial Theory and Practice,* 2nd edition (Boca Raton: Chapman & Hall).

Bowers, Newton L., Gerber, Hans U., Hickman, James C., Jones, Donald A., and Nesbitt, Cecil J. (1997) *Actuarial Mathematics* (Illinois: The Society of Actuaries).

Brown, Jeffrey R., Olivia S. Mitchell, James M. Poterba, and Mark J. Warshawsky (2001). *The Role of Annuity Markets in Financing Retirement*, (Cambridge: MIT Press).

Cairns, A.J.G., Blake, D. and Dowd, K. (2006) "A two-factor model for stochastic mortality with uncertainty: theory and calibration." *Journal of Risk and Insurance, 73(4)*, pp.687-718.

Cairns, A.J.G., Blake, D., Dowd, K., Coughlan, G.D., Epstein, D., Ong, A., and Balevich, I. (2009) "A quantitative comparison of stochastic mortality models using data from England & Wales and the United States." *North American Actuarial Journal, 13(1),* pp.1-35

Cannon, E., and Tonks, I. (2004) "U.K. Annuity Rates, Money's worth and Pension Replacement Ratios, 1957-2002" *The Geneva Papers on Risk and Insurance - Issues and Practice 29(3)*, pp.371-393.

Cannon, E., and Tonks, I. (2008) *Annuity Markets* (Oxford: Oxford University Press).

Cannon, E., and Tonks, I. (2009) "Money's worth of pension annuities." Dept for Work and Pensions Research Report No 563.

Cannon, E., Stevens, R., and Tonks, I. (2012) "Price Efficiency in the Dutch Annuity Market." University of Bristol mimeo.

CMI (2002) "An interim basis for adjusting the '92' series mortality projections for cohort effects." Continuous Mortality Investigation Working Paper 1.

CMI (2006) "Stochastic projection methodologies: further progress and P-spline model features, example results and implications." Continuous Mortality Investigation Working Paper 20.

CMI (2009) "A prototype mortality projections model: part one – an outline of the proposed approach." Continuous Mortality Investigation Working Paper 38.

De Meza, D., and Webb, D. (2012) "Correlation-Based Tests for Asymmetric Information in Insurance Markets are Uninformative" LSE mimeo.

Debt Management Office (2013) *GEMM Guidebook: A guide to the roles of the DMO and Primary Dealers in the UK government bond market*.

Financial Services Authority (2007) "Annuitant longevity improvements" FSA Dear CEO Letter, April.

Finkelstein, Amy, and James M. Poterba, (2002). 'Selection Effects in the UK Individual Annuities Market' *Economic Journal, 112(476):* pp.28-50.

Finkelstein, Amy, and James M. Poterba, (2004). 'Adverse Selection in Insurance Markets: Policyholder Evidence from the U.K. Annuity Market' *Journal of Political Economy, 112(1):* pp.183-208.

Joelle HY. Fong, Jean Lemaire, Yiu Kuen Tse (2011) "Improving Money's Worth Ratio Calculations: the Case of Singapore's Pension Annuities" SSRN working paper 1928323.

Fong, Joelle H. Y.1; Mitchell, Olivia S.2; Koh, Benedict S. K.3 (2011) "Capital Accumulation and Uncertain Lifetimes With Adverse Selection" *Journal of Risk and Insurance, 78(4)*, pp. 961-982.

Girosi, F., and King, G. (2008) *Demographic Forecasting* (Princeton: Princeton University Press).

H.M.Treasury (2010a) S*pending Review* (London: The Stationery Office).

HM Treasury (2010b) *Removing the Requirement to Annuitise by age 75*: a summary of the consultation responses and the Government's response, HM Treasury, London.

HM Treasury and Bank of England (1995) *Report of the Debt Management Review.*

Howard, R.C.W. (2006) *Canadian Annuitant Mortality Table*. (CIP 2005).

Lee, R.D., and Carter, L.R. (1992) "Modelling and forecasting U.S. mortality" *Journal of the American Statistical Association, 87(419)*, pp.659-75.

Mitchell, Olivia S., Poterba, James M., Warshawsky, Mark J. and Brown Jeffrey R. (1999) "New Evidence on the Money's Worth of Individual Annuities," *American Economic Review, 89*, pp.1299-1318. This article is reprinted in Brown et al (2001).

Parliamentary and Health Service Ombudsman (2008) *Equitable Life: a decade of regulatory failure* (London: The Stationery Office).

Penrose (2004) *Report of the Equitable Life Inquiry* (London: The Stationery Office).

Pitacco, E., Denuit, M., Haberman, S., and Olivieri, A. (2009) *Modelling Longevity Dynamics for Pensions and Annuity Business* (Oxford: Oxford University Press).

Plat, R. (2009) "On stochastic mortality modeling." Pensions Institute Discussion Paper PI-0908.

Rothschild, M., & Stiglitz, J. (1976) "Equilibrium in competitive insurance markets: An essay on the economics of imperfect information." *Quarterly Journal of Economics*, 90(4), pp.629-649.

Stark, J. (2002) "Annuities: the consumer experience", *ABI Research Report*, (ISBN 1-903193-22-2).

Telford, P.G., Browne, B.A., Collinge, P., Fulchar, P., Johnson, B.E., Little, W., Lu J.L.C., Nurse, J.M., Smith, D.W. and Zhang, F. (2011) "Developments in the Management of Annuity Business" *British Actuarial Journal,* Vol. 16, No. 3, 471-551.

Warshawsky, M. (1988) 'Private Annuity Markets in the United States: 1919-1984' *The Journal of Risk and Insurance, Vol. 55, No. 3*, 518-528.

Figures and graphs

Figure 1 Mortality assumptions of life assurers

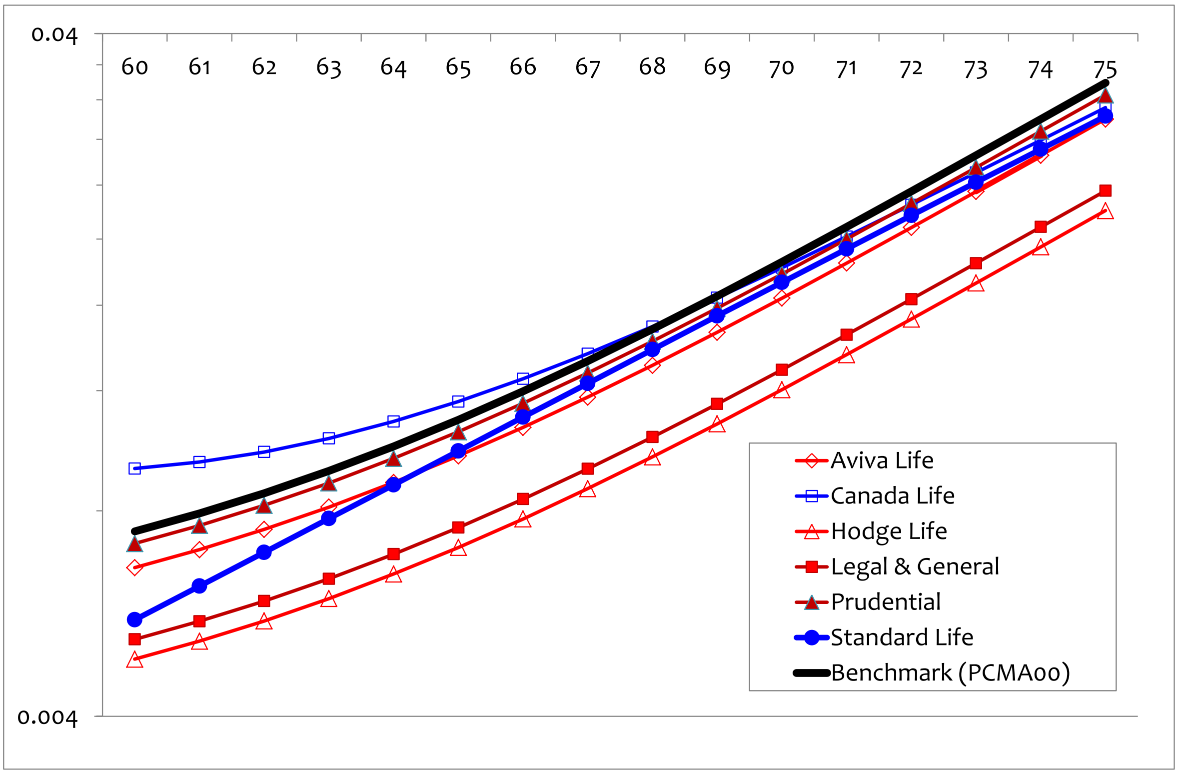


Figure 2 UK Annuity Rates (Male, Compulsory Purchase) and Bond Yields

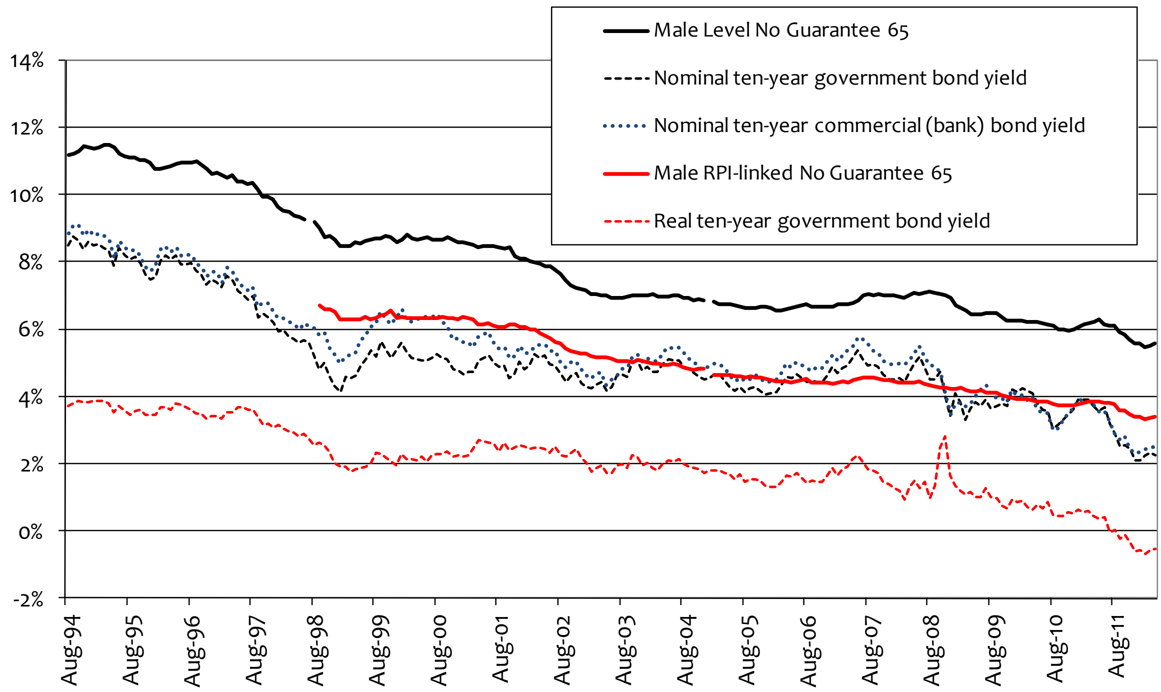
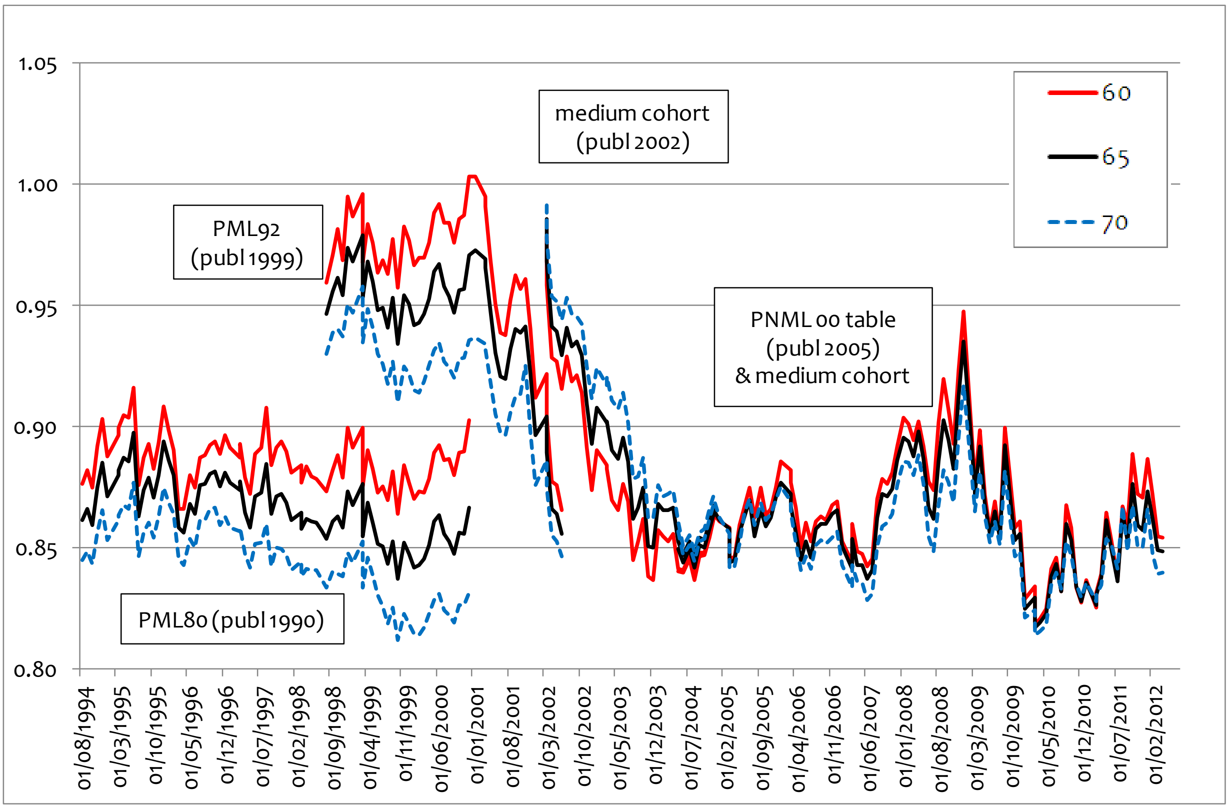


Figure 3: Money's worth calculations, level annuities for different ages



PML80 refers to data from 1994-2001; PML92 refers to data from 1999-2002; medium cohort refers to data from 2002-2005; and PNML00 refers to data from 2005 to 2012.

Figure 4: Money's worth calculations, different guarantee periods, male, 65

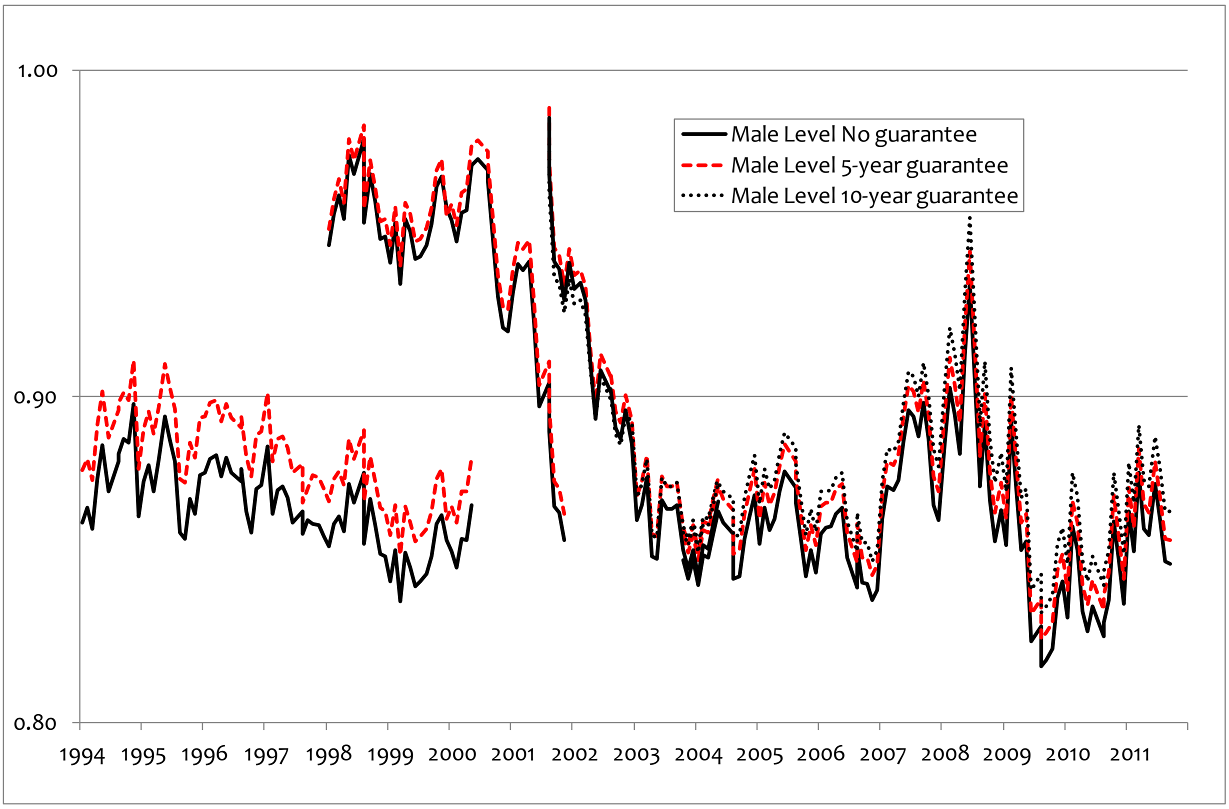


Figure 5: Money's worth calculations, different types of annuity, male 65

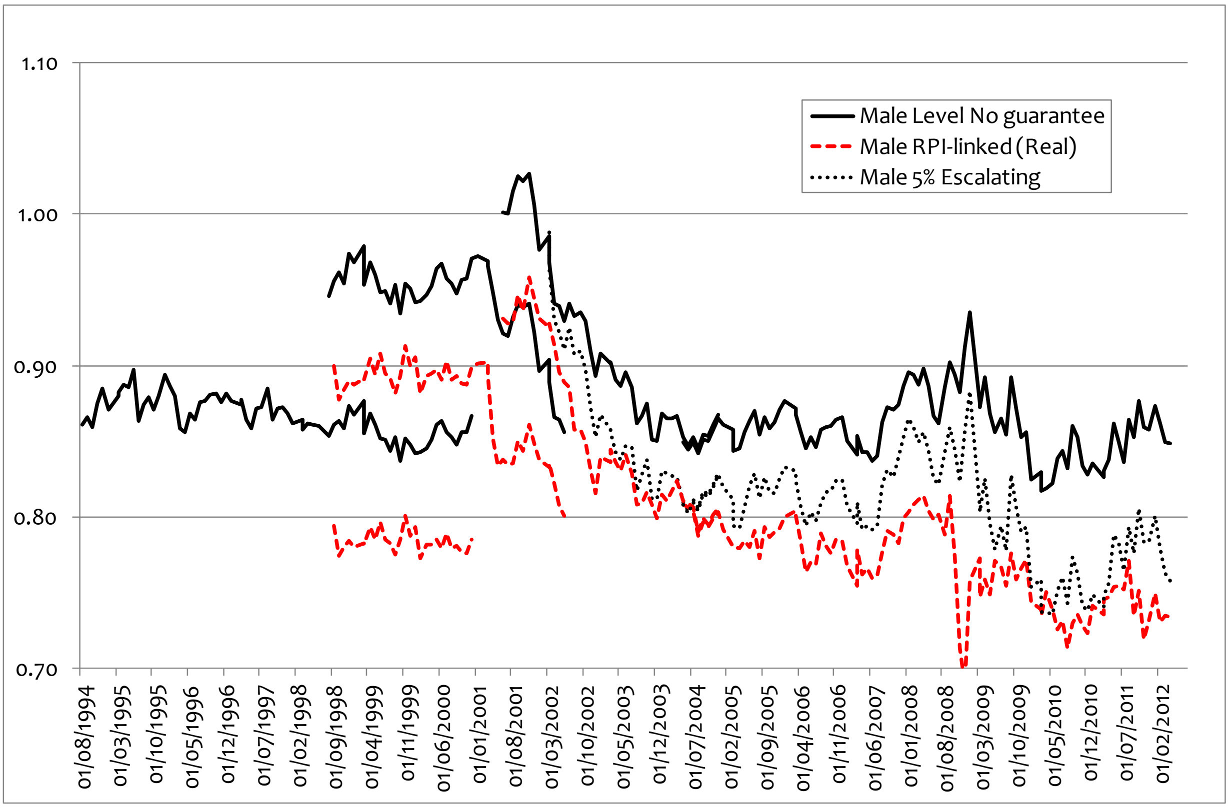


Figure 6: Diagram of separating equilibrium

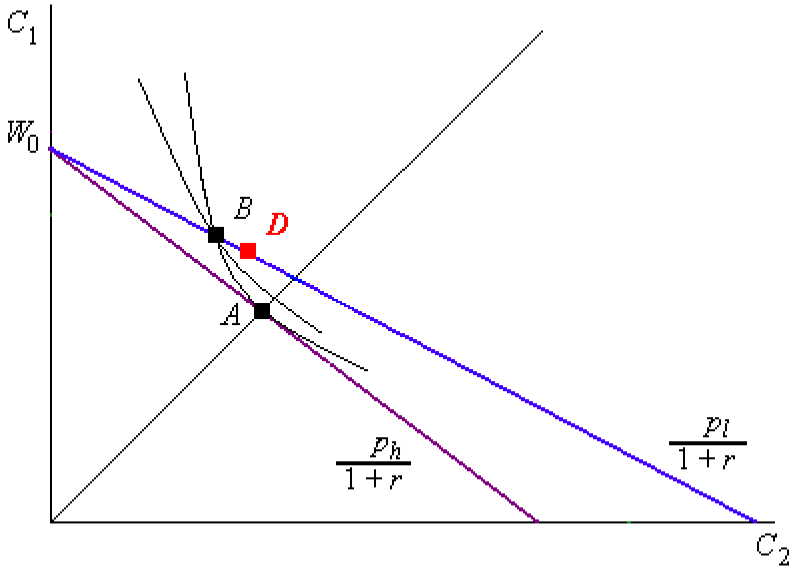


Figure 7: Fan chart of survival probabilities, male 65

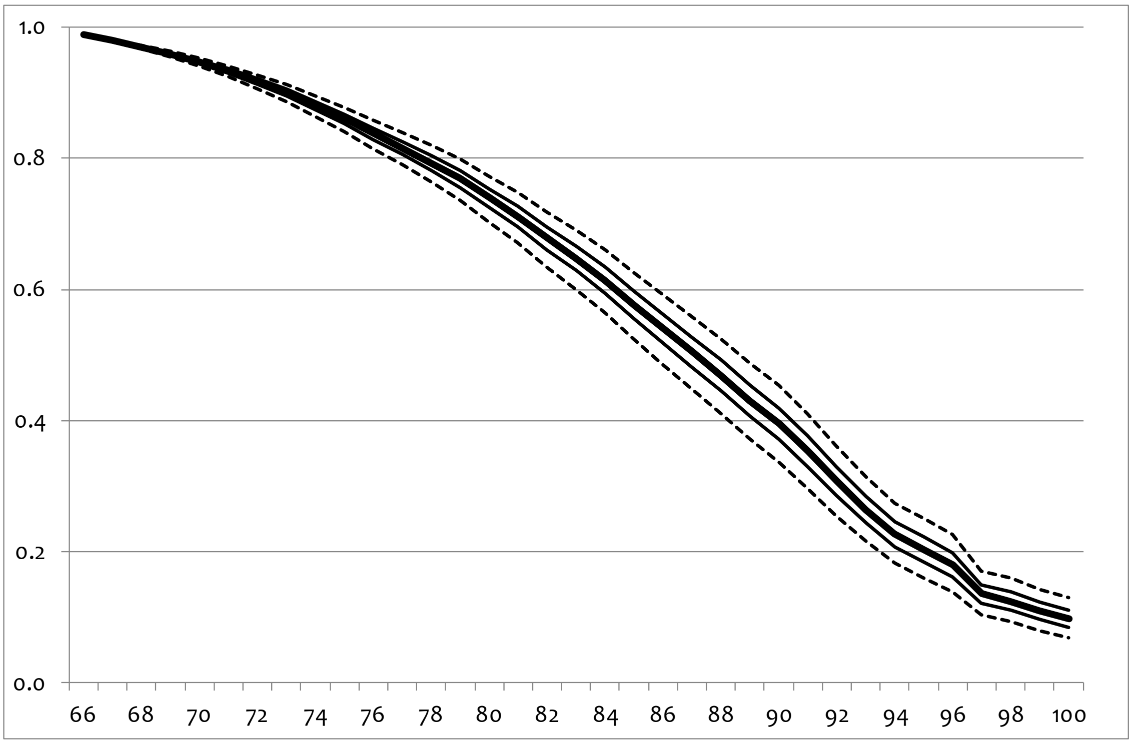


Figure 8: Annuity Value Distributions, male 65

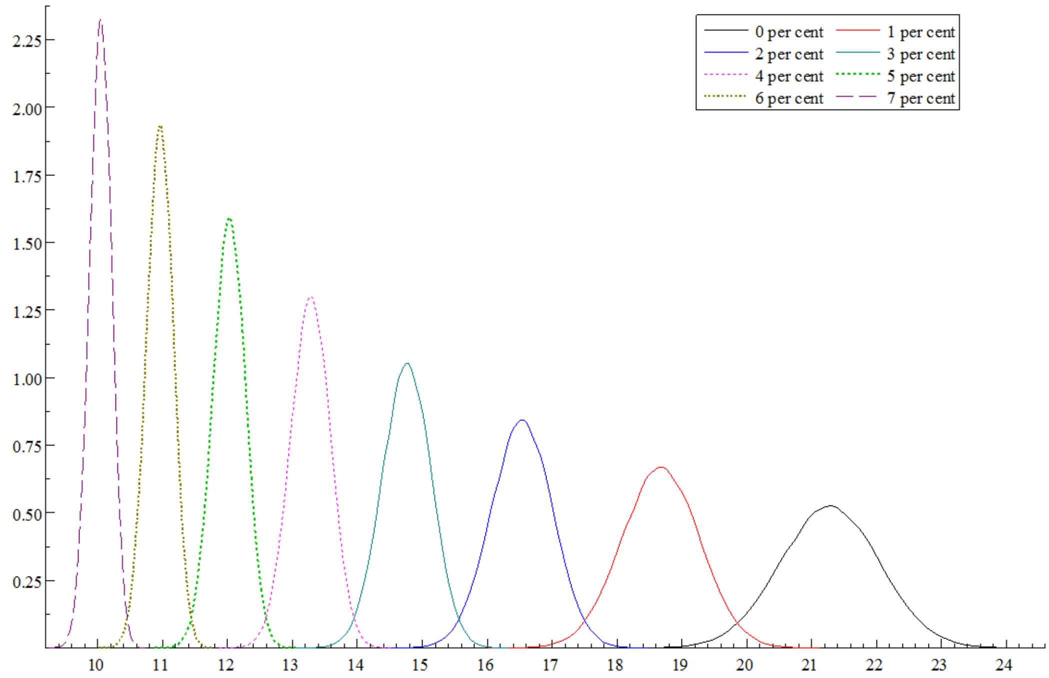
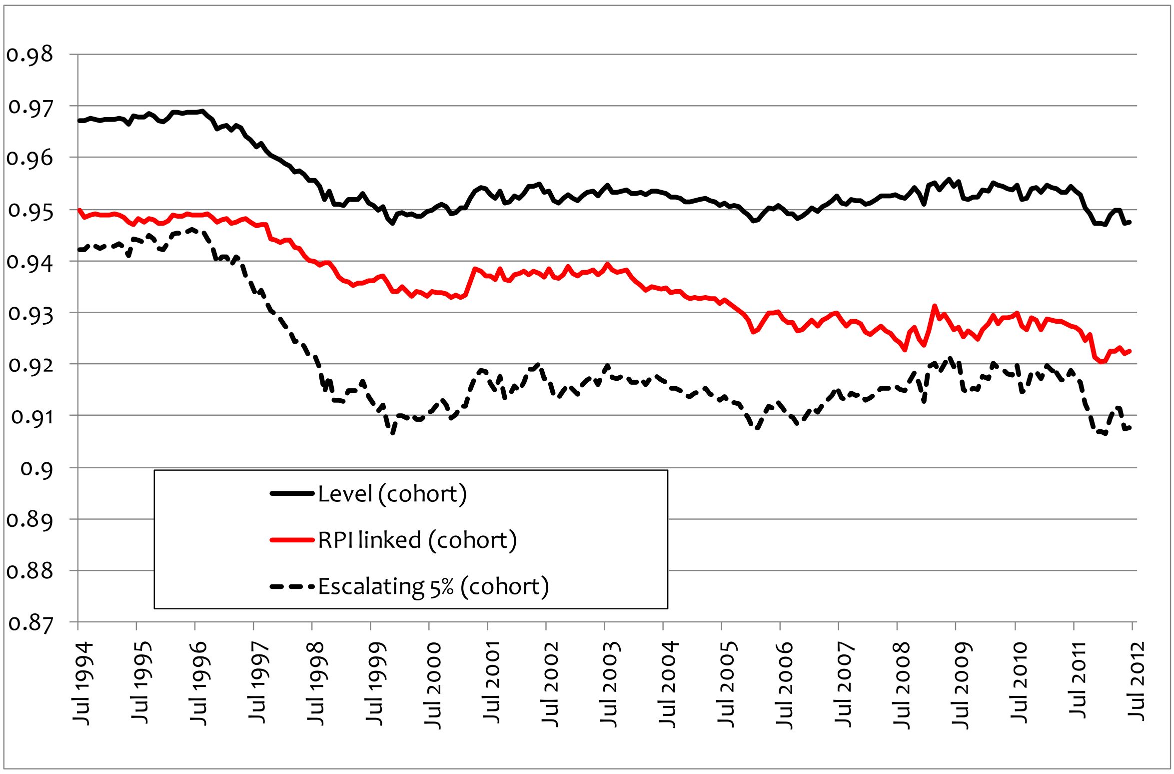


Figure 9: Money's worths using actual yields



Tables

Table 1: Summary of mortality assumptions in the FSA returns

Company	Mortality assumption
Aviva Life	88.5% of PCMA00
Canada Life	89% of RMV00 (plus further adjustments)
Hodge Life	65% of PCMA00
Legal and General	69.5% of PCMA00 (plus further adjustments)
Prudential	95% of PCMA00
Standard Life	88.4% of RMC00

Table 2: Purchases of different annuity types, 2011

Source: various FSA Returns, Appendix 9.3, Form 47, rows 400 and 905

Company	Nominal		Real	
	No of purchases	Average purchase	No. of purchases	Average purchase
Aviva Annuities	58,692	£32,696	1521	£33,707
Canada Life	13,440	£32,696	909	£72,939
Hodge Life	859	£31,029	none	
Legal & General	25,928	£25,102	845	£18,073
Prudential	37,006	£25,653	307	£33,707
Standard Life	20,361	£14,844	245	£20,804

Table 3: Testing for Differences in Money's worths by age, and product type

		Sept. 1994-Dec. 2000: 80 Life-Table	t-test on "matched pair differences"	*Sept. 1998-Dec. 2003: 92 Life-Table*	t-test on "matched pair differences"	*Jan. 2001-Dec. 2004: medium Life-Table*	t-test on "matched pair differences"	*May 2004-Apr. 2012: oo Life-Table*	t-test on "matched pair differences"
Panel A: Different Ages									
Level, NG, male 65	*Obs.*	77		65		48		96	
	Mean	0.866	*Base-case*	0.909	*Base-case*	0.927	*Base-case*	0.859	*Base-case*
	St.dev	0.013		0.061		0.069		0.021	
Level , NG, male 70	*Obs.*	77		65		48		96	
	Mean	0.845	12.42***	0.889	6.15***	0.933	-1.94*	0.854	4.16***
	St.dev	0.016		0.053		0.063		0.018	
Level , NG, male 75	*Obs.*	41		65		48		96	
	Mean	0.812	15.18***	0.872	6.05***	0.925	0.2	0.850	4.43***
	St.dev	0.014		0.046		0.052		0.017	
Panel B: Different Guarantees									
Level , 5-year guarantees, male 65	*Obs.*	77		65		48		96	
	Mean	0.881	-29.00***	0.915	-10.67***	0.932	-6.39***	0.867	-48.42***
	St.dev	0.014		0.059		0.067		0.021	
Level ,10-year guarantees, male aged 65	*Obs.*	0		23		35		96	
	Mean			0.847	-7.02***	0.893	-0.81	0.873	-17.80***
	St.dev			0.029		0.032		0.022	
Panel C: Different products									
Real (RPI-linked), NG, male 65	*Obs.*	28		64		48		96	
	Mean	0.784	18.61***	0.840	18.57***	0.867	16.11***	0.768	15.58***
	St.dev	0.007		0.063		0.064		0.027	
Escalating 5%, NG, male 65	*Obs.*	0		23		35		96	
	Mean			0.770	11.50***	0.856	7.05***	0.802	10.14***
	St.dev			0.042		0.048		0.033	

The t-test on "matched pair differences" compares the money's worth of the relevant annuity product with the base-case of the equivalent level annuities NG, male aged 65. The standard errors for these tests are Newey-West st. errors with 10 lags. Where NG denotes no guarantees; *, **, *** denotes significance at 90, 95 and 99 per cent respectively.

Table 4: Stochastic Money's Worth Calculations

	Diversification (many policies)				Imperfect diversification (400 policies)			
Quantile:	**0.50**	**0.90**	**0.95**	**0.99**	**0.5**	**0.9**	**0.95**	**0.99**
Interest rate								
-1%	1.000	0.933	0.915	0.884	1.001	0.933	0.914	0.879
0%	1.000	0.940	0.924	0.896	1.001	0.936	0.919	0.886
1%	1.000	0.946	0.932	0.906	1.001	0.939	0.923	0.891
2%	1.000	0.952	0.939	0.916	1.001	0.942	0.927	0.897
3%	0.999	0.956	0.945	0.924	1.001	0.945	0.930	0.902
4%	0.999	0.961	0.950	0.932	1.000	0.948	0.934	0.907
5%	0.999	0.964	0.955	0.938	1.000	0.950	0.937	0.912
6%	0.999	0.968	0.959	0.944	1.000	0.953	0.940	0.916
7%	0.999	0.971	0.963	0.949	1.000	0.955	0.943	0.920
8%	0.999	0.973	0.966	0.954	1.000	0.957	0.945	0.924
9%	0.999	0.976	0.969	0.958	1.000	0.959	0.948	0.927
10%	0.999	0.978	0.972	0.961	1.000	0.961	0.950	0.930

Table shows the ratio of the relevant quantile of the annuity distribution to the mean, from equation (9). The projection is made from the Lee-Carter model, assuming parameter uncertainty. The left-hand panel assumes that there are sufficiently many policies that the only risk is from projecting the cohort mortality: the right-hand panel combines cohort mortality risk with additional risk from having only 400 policy holders.

Appendix A: Description of the data

Data on UK annuity rates for males and females at various ages are taken from MoneyFacts over the period August 1994 to April 2012 and update the annuity series that we have published previously: the construction of these series is described in more detail in Cannon and Tonks (2008, 2010). These are compulsory-purchase annuities which are bought as part of a pension scheme (where tax relief has been given on the accumulation of the pension fund via an EET scheme).

From about 2011 some life assurers started to price annuities based on the postcode of the annuitant (life expectancy varies by region and postcode has considerable predictive power): where life assurers did this the annuity rate data we have is for a "typical" postcode – but the definition of "typical" is decided by the life assurer and we do not know what definition is used. Notice further that from December 2012 it became impossible to price annuities for men and women differently under the ECJ directive, so there would be little point in extending our analysis to later time periods as there will be further changes to the market due to unisex pricing.

In Figure 1 we illustrate the annuity rate series for a 65-year old male over time compared with government bond data, and summary statistics of this data for nominal and real variables is presented in Tables A1 and A2. All bond data is taken from the Bank of England web-site. It can be seen that nominal annuities approximately track the nominal bond yield and analogously for real annuities: annuity rates are highly correlated with long-term bond yields, and the average difference in these two series over the sample period was 2.86%. We also compare the two sub-periods up to the financial crisis (Northern Rock bank run in August 2007) and since the onset of the crisis. Since the crisis, both short-term (base rate) and long-term government bond yields have fallen, and this has been reflected in a fall in annuity rates. Real annuities have payments that rise in line with the UK's Retail Price Index.

Table A1: Monthly Time Series Properties of Nominal Pension Annuity for 65-year old males and various alternative bond yields

	Annuity Rate for 65-year old males	Long-term: 10 year Government Bond Yield	Short-term: Bank of England Base rate	Interest rates on retail term deposits	Difference in Annuity Rate and Government Bond Yield
Panel A: Aug 1994 – April 2012					
Mean	7.96%	5.10%	4.43%	3.21%	2.86%
St.Dev.	1.70%	1.49%	2.09%	1.54%	
Correlation	0.93				
Panel B: Aug 1994 – July 2007					
Mean	8.54%	5.59%	5.34%	3.81%	2.95%
St. Dev.	1.63%	1.38%	1.07%	1.18%	
Correlation	0.92				
Panel C: Aug 2007 – Apr 2012					
Mean	6.40%	3.77%	1.88%	1.62%	2.63%
St. Dev.	0.49%	0.79%	2.12%	1.23%	
Correlation	0.88				

Table 1a presents descriptive statistics on the monthly time series of average annuity rates in the CPA market, long-term and short-term government bond yields and rates on retail term deposits, over the period 1994 to 2012 and for the two sub-periods.

Table A2: Monthly Time Series Properties of Real Pension Annuity for 65-year old males and various alternative bond yields

	RPI-linked Annuity Rate for 65-year old males	Long-term: 10 year Real Government Bond Yield	Difference in Real Annuity Rate and Real Government Bond Yield
Panel A: Sept 1998 – April 2012			
Mean	4.93%	1.60%	3.34%
St.Dev.	0.95%	0.77%	
Correlation	0.81		
Panel B: Sept 1998 – July 2007			
Mean	5.43%	2.02%	3.41%
St. Dev.	0.78%	0.35%	
Correlation	0.71		
Panel C: Aug 2007 – Apr 2012			
Mean	4.01%	0.80%	3.20%
St. Dev.	0.34%	0.73%	
Correlation	0.88		

Table 1b presents descriptive statistics on the monthly time series of average real annuity rates in the CPA market and real long-term government bond yields over the period 1994 to 2012 and for the two sub-periods.

The remaining data that we need to estimate the money's worth are the mortality projections. In previous money's worth calculations (Cannon and Tonks, 2004, 2008, 2009) we attempted to update the mortality projections at the time that the life assurers did so. We did not try to infer the mortality tables used by the life assurers from the footnotes of the FSA Returns because of a variety of problems: each FSA Return contains a variety of assumptions; there are a large number of companies; and until recently the footnotes of the FSA Returns were not easily available. Instead we started using each new table from the Institute of Actuaries from a year before the publication date, on the argument that the broad outline of these data may have been known to life assurers before actual publication (and life assurers would also have been able to analyse the mortality experience of their own annuitants).

The PML80 ("Purchased Male Life") table was published in 1992 ("80" refers to the base year). Although it projected gradual increases in life expectancy, by the late 1990s it had become clear that the downward trend in mortality of pensioners was much stronger and the PML92 tables (published 1999) revised life expectancy up by almost two years. Further analysis of the reduction in mortality both for pensioners and people of below pension age (for which pension data were unavailable: life assurance data was used instead), suggested a "cohort" effect, ie a discrete downward jump in mortality for people born after about 1930. This led to a set or "interim adjustments" published in 2002: the most widely used "medium cohort" adjustment is illustrated here. In 2005 information on the most recent annuitant mortality was published (the "00" table), which did not have an accompanying projection for changes into the future. Accordingly at that time many life assurers used the "00" table as a base and then used the "medium cohort" projection from 2000 (or some other year) onwards.

Appendix B: Implementation of the Lee-Carter method

Lee and Carter (1992) introduced this model and good expositions are Girosi & King (2008, pp.34.ff) and Pitacco et al (2008, pp.169-173 & 186.ff.). This specification does not completely identify the parameters, so identifying restrictions (which have no effect on the analysis) are used:

$$\sum_t \kappa_t = 0 \quad \text{and either}$$
$$\sum_x \beta_x = 1, \quad \text{in the original Lee-Carter (1992) paper}$$
$$\text{or} \quad \sum_x \beta_x^2 = 1, \quad \text{in Girosi-King (2008)}$$

Assuming that the errors are Normally distributed, then estimation is as follows (taken from GK, but with slightly different notation):

$$x = \{1, \dots, X\} \quad \text{nb slightly different from typical actuarial notation:}$$
$$\text{typically start age age 60, so } x \equiv age - 59$$
$$t = \{1, \dots T\}$$
$$\mathbf{M} = \left(m_{x,t}\right) \in \mathbb{R}^X \times \mathbb{R}^T \quad \text{nb: ages in rows, time in columns}$$
$$\tilde{\mathbf{M}} \equiv \mathbf{M} - \bar{\mathbf{m}}_x \mathbf{1}' \qquad \bar{\mathbf{m}}_x = \{\bar{m}_x\} \in \mathbb{R}^X$$

The estimation of the intercept term is straight-forward and intuitive: given the constraint that $\sum_t \kappa_t = 0$, just take the row means to get

$$\hat{\alpha}_x = \bar{m}_x = T^{-1} \sum_{t=0}^{T-1} m_{xt}$$

Consider a model with l principal components:

$$m_{xt} = \alpha_x + \kappa_{1t}\beta_1 + \cdots + \kappa_{lt}\beta_l + \varepsilon_t$$
$$\beta_i \in \mathbb{R}^X$$
$$\underset{X \times T}{\tilde{M}} = \underset{X \times l}{\beta}\ \underset{l \times T}{\kappa} + \varepsilon_t$$

By the singular-value decomposition theorem

$$\underset{X \times T}{\tilde{\mathbf{M}}} = \underset{X \times T}{\mathbf{B}}\ \underset{T \times T}{\mathbf{L}}\ \underset{T \times T}{\mathbf{U}'}$$

where **L** is a diagonal matrix with the singular values put in descending order. The first l principal components are the first l columns of **B**. The Lee-Carter model is just this model with $l = 1$. So

$$\text{Lee-Carter} \begin{cases} \hat{\beta} = B_1 / 1' B_1 \\ \hat{\kappa} = (1' B_1) L_{11} U_1' \end{cases} \qquad \text{Girosi-King} \begin{cases} \hat{\beta} = B_1 / \|B_1\| \\ \hat{\kappa} = \hat{\beta}' \tilde{M} \end{cases}$$

Lee and Carter suggest an adjustment to the time-effects to ensure that the expected deaths match the actual deaths, called second-stage estimation, so that

$$\forall t : \kappa_t \Leftarrow 0 = \sum_x ETR_{xt} e^{\bar{m}_x + \beta_x \kappa_t} - \sum_x D_{xt}$$

and we have followed that procedure.

Our estimates of the parameters are shown in the figures below.

Figure A1: Lee Carter Parameters for the UK Pensioner Data